Birzeit University
Department of Physics
Quantum Mechanics Phys635
Fall 2018
First Exam, Dec. 11th 2018

1. Consider a particle in three-dimensional space, whose state vector is $|\psi>$ and whose wave function is $\psi(r) =< r|\psi >$. Let A be an observable which commutes with $\vec{L} = \vec{R} \times \vec{P}$, the orbital angular momentum of the particle. Assuming that A, L^2 and L_z form a set of commuting observables let $|n, l, m>$ their common eigenkets, whose eigenvalues are, respectively, a_n, $l(l+1)\hbar^2$ and $m\hbar$ (the index n is assumed to be discrete).

   Let $U(\phi)$ be the unitary operator defined by:

   $$U(\phi) = e^{\frac{-iL_z\phi}{\hbar}}$$

   where ϕ is a real dimensionless parameter. For an arbitrary operator K, we call $\tilde{K}$ the transform of K by the unitary operator $U(\phi)$:

   $$\tilde{K} = U(\phi)KU^{\dagger}(\phi)$$

   (a) (15 points) We set $L_+ = L_x + iL_y$, $L_- = L_x - iL_y$. Calculate $\tilde{L_+}|n, l, m>$ and show that L_+ and $\tilde{L_+}$ are proportional; calculate the proportionality constant.

   (b) (15 points) Express L_x terms of $\tilde{L_x}$, $\tilde{L_y}$,and $\tilde{L_z}$,. What geometrical transformation can be associated with the transformation of L into $\tilde{L}$

2. (20 points) Show that the Clebsch-Gordon coefficients are the matrix element of a unitary matrix that change the basis from $|j_1 j_2, m_1 m_2>$ to $|JM>$

3. A particle of spin S =3/2.

   (a) (15 points) Write the matrix representation for S_x, S_y, S_z and S^2

   (b) (10 points) let $|\psi(t=0)> = \frac{1}{2\sqrt{2}}(|\frac{3}{2}\frac{3}{2}> + \sqrt{3}|\frac{3}{2}\frac{1}{2}> + \sqrt{3}|\frac{3}{2}\frac{1}{2}> + |\frac{3}{2}\frac{-3}{2}>)$ Show that it is an eigenvector of S_x.

   (c) (8 points) If the particle a magnetic moment $\vec{\mu} = g\vec{S}$, and is placed in uniform magnetic field that points in the x-direction. The particle initial wave-function is given in the previous part. Find $<\hat{S}_y>(t)$

4. We define the standard components of a vector operator V as the three operators:

   $$V_1^{(1)} = -\frac{1}{\sqrt{2}}(V_x + iV_y)$$
   $$V_0^{(1)} = V_z$$
   $$V_{-1}^{(1)} = \frac{1}{\sqrt{2}}(V_x - iV_y)$$

   Using the standard components $V_p^{(1)}$ and $W_q^{(1)}$ of the two vector operators V and W, we construct the operators:

   $$[V^{(1)} \otimes W^{(1)}]_M^{(K)} = \sum_p \sum_q < 11; pq|KM > V_p^{(1)} W_q^{(1)}$$

   where the $< 1, 1; p, q|K, M >$ are the Clebsch-Gordan coefficients entering into the addition of two angular momenta 1

   (a) (10 points) Show that $[V^{(1)} \otimes W^{(1)}]_0^{(0)}$ is proportional to the scalar product $V \cdot W$ of the two vector operators.

(b) (10 points) Show that the three operators $[V^{(1)} \otimes W^{(1)}]^{(1)}_M$ are proportional to the three standard components of the vector operator $V \times W$

(c) (10 points) Express the five components $[V^{(1)} \otimes W^{(1)}]^{(2)}_M$ in terms of V_z, $V\pm = V_x \pm iV_y$, W_z, $W\pm = W_x \pm iW_y$

Good Luck

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Question:	1	2	3	4	Total
Points:	30	20	33	30	113
Score:					